Statistical modeling for spatial-temporal soil moisture data in Oklahoma

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1. Materials and Methods

The knowledge of soil moisture dynamics is crucial for understanding and modeling hydrological processes, like precipitation, infiltration, evapotranspiration, runoff, and drainage Dingman (1994), which are affected by soil property, relative humidity, change in air temperature and other factors. Also we suspect spatial pattern exists. So we use spatial-temporal methods to model the soil moisture and to find spatial- temporal relations between soil moisture and climate variables. We use the hourly-accumulations soil moisture and climate data of Oklahoma Mesonet (https://www.mesonet.org/). Considering the completeness of the data, we use the data of 2010 from 106 Mesonet stations.

2. The Model

Our model is still under investigation. For preliminary study we propose a spatial-temporal model Wikle & Hooten (2010) to model the data. The soil moisture is considered as a one-step Markov Chain over time at each site.
Observation Equation:

$$\boldsymbol{Z}(s;t) = \boldsymbol{Y}(s,t) + \boldsymbol{\epsilon}_{y,s,t} \quad \boldsymbol{\epsilon}_{y,s,t} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_0)$$

Process Equation:

$$\boldsymbol{Y}(s,t) = \boldsymbol{Y}(s,t-1) + \boldsymbol{X}(s,t)\boldsymbol{\beta} + \boldsymbol{Z}(s)\boldsymbol{\gamma} + \boldsymbol{\phi}(s,t) + \boldsymbol{e}(s,t)$$

where

$$\boldsymbol{e}(s,t) \sim N(0, \boldsymbol{\Sigma_s} \otimes \boldsymbol{\Sigma_\theta})$$

Here $\mathbf{Z}(s,t)$ is the observation vector corresponding to site s at time t; $\mathbf{Y}(s,t)$ is the state vector representing the hidden process. $\mathbf{X}(s,t)$ corresponds to climate process both in time and space and $\mathbf{Z}(s)$ is the soil property which is pure spatial process. (α, β) are regression coefficients. For now we assume normality in our error terms. Hence we attempt to decompose the process into a pure spatial process and a possible spatial-temporal process. Also we assume the error terms are constant over time, so we can write the error term for process equation as the Kronecker product of the covariance among variables and the variogram matrix. Also

$$\boldsymbol{\phi}(s,t) = \boldsymbol{M}\boldsymbol{\phi}(s,t-1) + \boldsymbol{V}(s)$$

where $\boldsymbol{\phi}(s,t)$ is a latent spatial-temporal process with covariance in Matern Gaussian form

$$\boldsymbol{V}(s) \sim GP(\mathbf{0}, \boldsymbol{Q})$$

A Markov Chain Monte-Carlo (MCMC) algorithm, Kalman Forward-filtering, backward-smoothing method West & Harrison (1997) will be applied to obtain the full conditional posterior distribution of the state vector $\mathbf{Y}(s,t)$ and all unknown parameters Θ.

3. Mesonet: MARE

These pictures give a general idea of the shape of some variables and possible relations between them.

Figure 1: *Hourly data of soil moisture and rain fall*

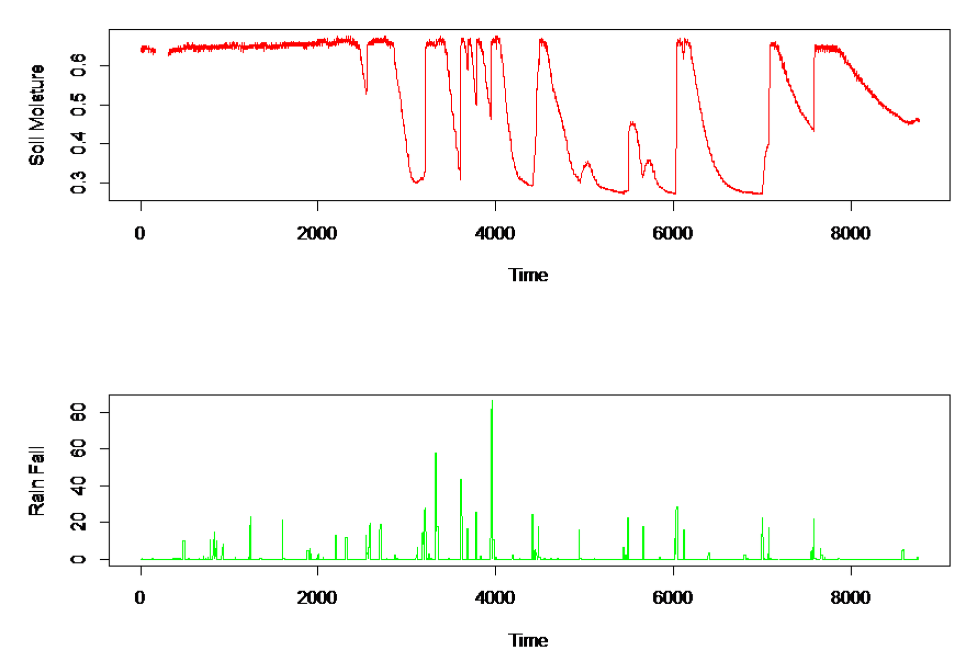


Figure 2: *Hourly data of relative humidity and air temperature*

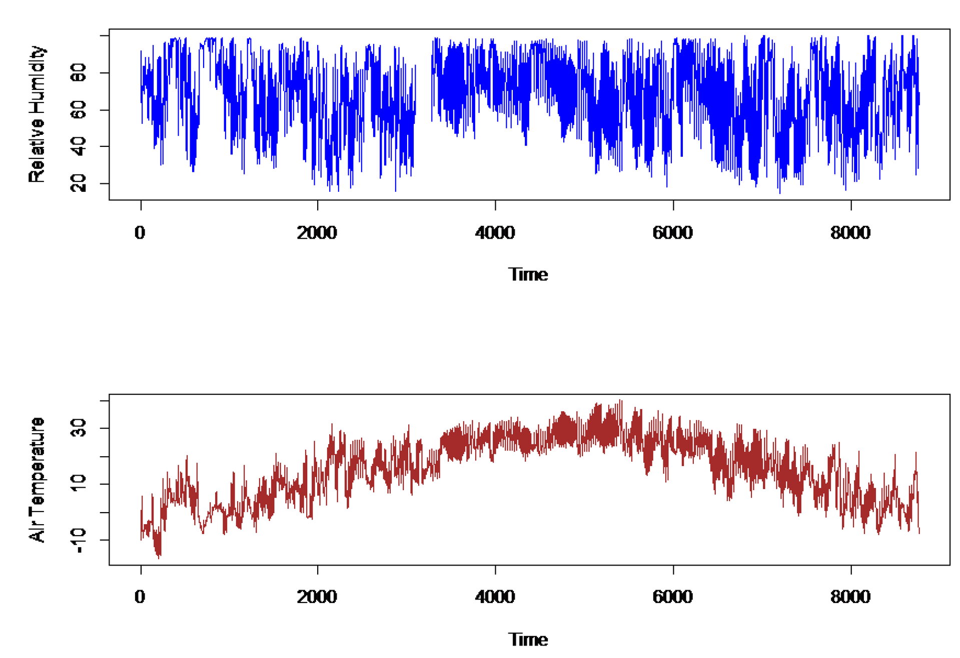
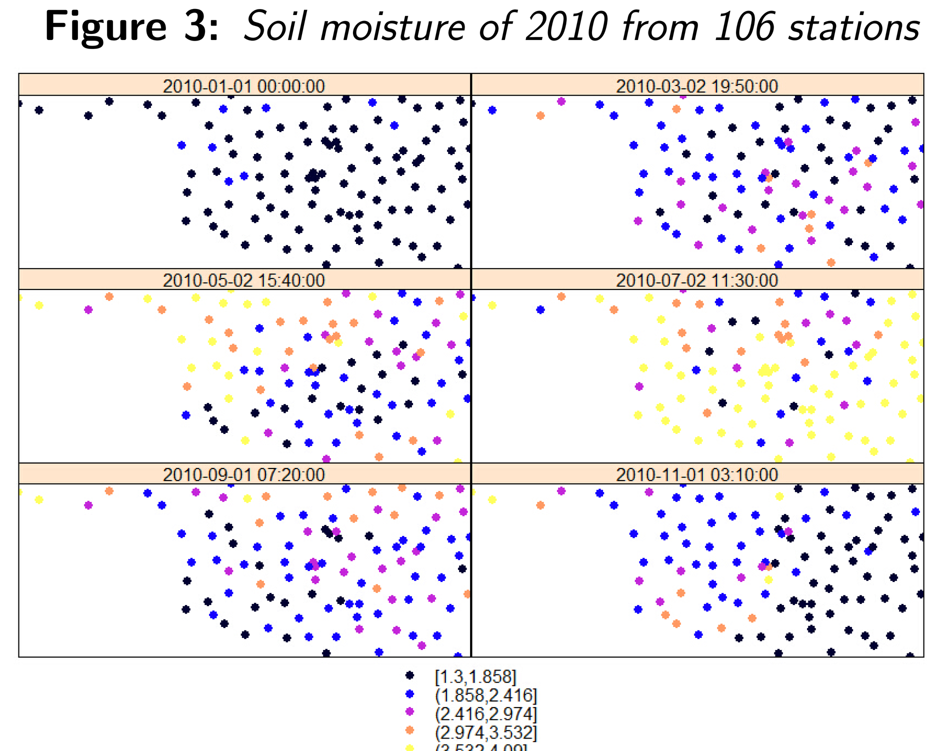
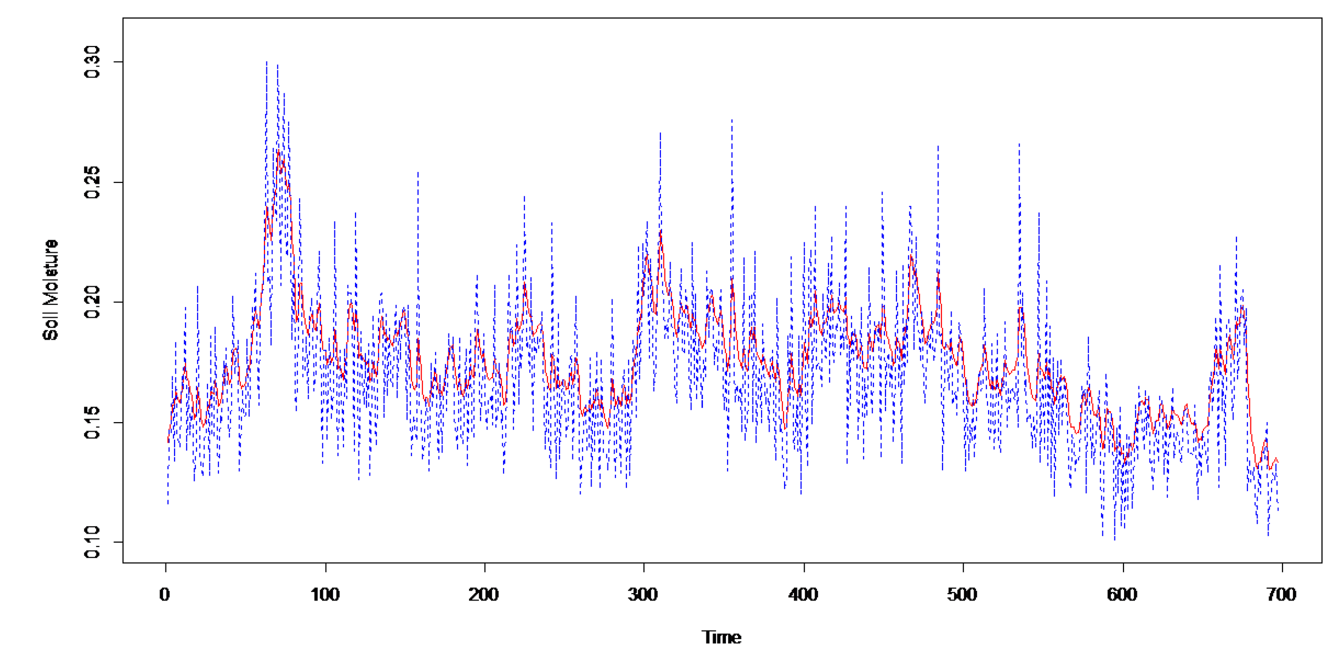


Figure 3: *Soil moisture of 2010 from 106 stations*



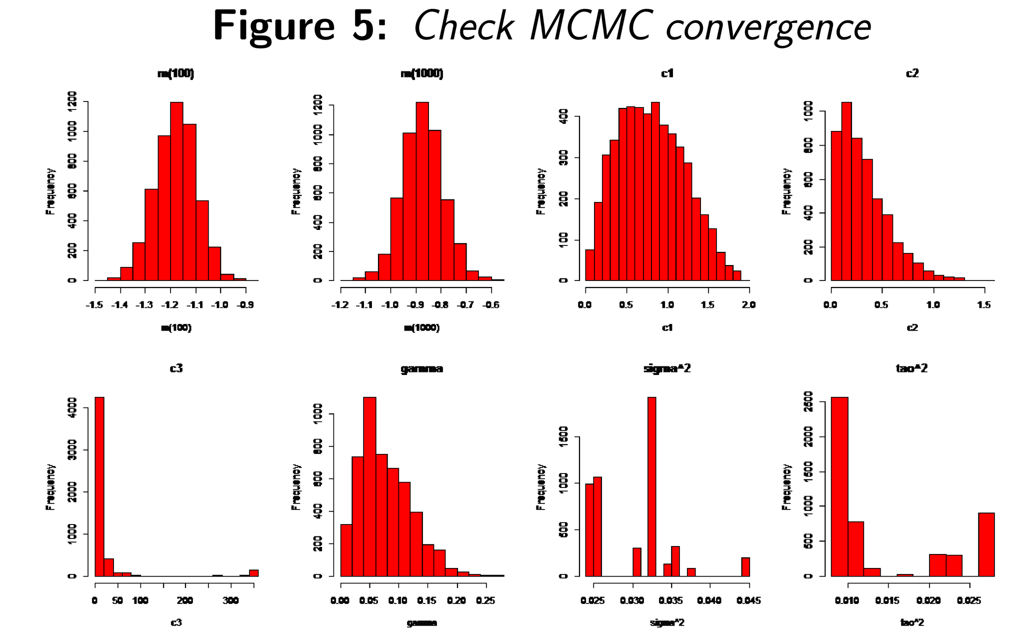
We also would like to introduce our previous work. We fit time series models using soil moisture data from COSMOS and precipitation data from Mesonet for two COSMOS sites. The picture below shows the model actually smoothed the raw data. We also estimated the parameters using Metroplis-Hastings algorithm.

Figure 4: *Estimates vs Raw data of Feb.2012*



The blue dash line represents the raw data. The red line represents the estimates by the model.
We also use this example to illustrate the convergence of the estimates of hyper-parameters.

Figure 5: *Check MCMC convergence*



4. Goal

We focus on the state-level soil moisture data from Mesonet. There are two aims in our analyses:
(1). Understanding temporal patterns of soil moisture in both a short term period and a long term period. We expect to establish a descriptive statistical model for changes over time;
(2). Understanding spatial variations of soil moisture at the state-level. We are going to use variables, such as climate and soil property, which may also vary in space and time, to explain the variation of soil moisture.
The goal is to predict soil moisture at those locations where data are not available in the entire state, i.e. a statistical mapping. Combining the knowledge of (1) and (2) will help us to establish a predictive model for mappings over time.

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References

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Wikle, C. K. & Hooten, M. B. (2010), 'A general science-based framework for dynamical spatio-temporal models', *Test* **19(3)**, 417–451.